Moduli spaces of abelian varieties

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In this course we will discuss some available methods and known results on the geometry of the moduli space $\mathcal{A}_g$ of principally polarized abelian varieties of dimension g.

Basic constructions

- The basics of birational geometry: ample, nef, effective cones of divisors; Kodaira dimension, minimal model
- Modular forms as sections of line bundles on $\mathcal{A}_g$; theta functions and the Satake compactification

The effective cone and the Kodaira dimension

- The Picard group of the partial compactification $\mathrm{Pic}(\mathcal{A}_g^{(1)})$ and the computation of the canonical class
- Modular forms of low slope; $\mathcal{A}_g$ is of general type for $g \geq 8$ (work of Tai, Freitag)
- Constructing geometric divisors on moduli spaces; Grothendieck-Riemann-Roch computations on the universal family; the Andreotti-Mayer divisor on the moduli spaces computation (work of Mumford); $\mathcal{A}_7$ is of general type

The ample cone and the minimal model

- The Picard groups of toroidal compactifications of $\mathcal{A}_g$
- The ample cone of the partial compactification (work of Hulek and Sankaran)
- The ample cone of the perfect cone toroidal compactification, and the canonical model of $\mathcal{A}_g$ (work of Shepherd-Barron)

The Chow and homology rings of $\mathcal{A}_g$

- The Chow and homology rings of $\bar{\mathcal{A}}_2$ and $\bar{\mathcal{A}}_3$ (work of van der Geer, Hain)
- Intersection numbers of divisors on $\bar{\mathcal{A}}_g$ (work of Erdenberger, G., and Hulek)

If time permits, we may also discuss the stable homology of $\mathcal{A}_g$ and of the Satake compactification; vector-valued modular forms; Andreotti-Mayer loci and the singularities of the theta divisor